This diagnostic test consists of two parts, **Fundamentals** and **Problem Solving**. If you can solve nearly all of the **Fundamentals** problems and at least half of the **Problem Solving** problems, then the Art of Problem Solving Online Class **Introduction to Algebra A** would only serve as a review for you. Answers to these problems are included at the end of this document. Do not use a calculator.

**Fundamentals**

1. **Solving Linear Equations.** Solve each of the following:
   
   (a) \(3x - 7 = 9 - x\)
   (b) \(\frac{7 - 2r}{3} = 4r\)

2. **Proportion.**
   
   (a) If \(x\) and \(y\) are directly proportional and \(x = 8\) when \(y = 20\), then what is \(y\) when \(x = 40\)?
   (b) If \(x\) and \(y\) are inversely proportional and \(x = 8\) when \(y = 20\), then what is \(y\) when \(x = 40\)?

3. **Systems of Equations.**
   
   (a) Find all values of \(r\) and \(s\) such that \(r - s = 5\) and \(3r - 5s = 9\).
   (b) Find all values of \(p\) and \(q\) such that \(3p + 7q = 1\) and \(5p = 14q + 53\).

4. **Graphing Lines.**
   
   (a) What is the slope of the line through \((3, 4)\) and \((-1, 3)\)?
   (b) What is the slope of the graph of \(3x - 4y = 7\)?
   (c) Find the slope-intercept form of the equation of the line through \((5, -2)\) and \((-1, 6)\).

5. **Introductory Quadratics.** Find all solutions to the following equations:
   
   (a) \(4x^2 = 81\)
   (b) \(x^2 + 8x + 12 = 0\)
   (c) \(x^2 - 3x - 88 = 0\)
Problem Solving

6. A box containing 3 oranges, 2 apples, and one banana weighs 15 units. Another box containing 5 oranges, 7 apples, and 2 bananas weighs 44 units. A third box containing 1 orange, 3 apples, and 5 bananas weighs 26 units. How much does each fruit weigh?

7. Find \( z \) if \( \frac{3}{1 - \frac{z}{2}} = 3z \).

8. At a certain time, Janice notices that her digital watch reads \( a \) minutes after two o’clock. Fifteen minutes later, it reads \( b \) minutes after three o’clock. She is amused to note that \( a \) is six times the value of \( b \). What time was it when she looked at her watch for the second time?

9. Calvin and Susie are running for class president. Of the first 80% of the ballots that are counted, Susie receives 53% of the votes and Calvin receives 47%. At least what percentage of the remaining votes must Calvin receive to catch up to Susie in the election?

10. The golden ratio is the largest number \( g \) such that \( \frac{g}{g + 1} = \frac{1}{g} \). Find \( g(g - 1) \).

11. A line passing through the points \((2a + 2, 3a^2)\) and \((3a + 4, 5a^2)\) has slope \( a + 3 \). Find all possible values of \( a \).

12. Describe all values of \( x \) that satisfy \( 7 - 3x < x - 1 \leq 2x + 9 \).
The answers to Do You Know Introduction to Algebra A are below. (The answers to problem sets and challenges given in the class will include full detailed solutions as opposed to the mere answers provided below.)

1. (a) $x = 4$, (b) $r = \frac{1}{2}$
2. (a) 100, (b) 4
3. (a) $(r, s) = (8, 3)$, (b) $(p, q) = (5, -2)$
4. (a) $\frac{1}{4}$, (b) $\frac{3}{4}$, (c) $y = -\frac{4}{3}x + \frac{14}{3}$
5. (a) $x = \frac{9}{2}, x = -\frac{9}{2}$; (b) $x = -2, x = -6$; (c) $x = 11, x = -8$
6. Oranges weigh 1 unit, apples weigh 5 units, and bananas weigh 2 units.
7. 3
8. 3:09
9. 62%
10. 1
11. $-1, 6$
12. $x > 2$