Do You Know?  

*Introduction to Geometry*, by R. Rusczyk

If you can solve nearly all of the following problems with little difficulty, then the text *Introduction to Geometry* would only serve as a review for you.

1. Prove the Pythagorean Theorem.
2. Find $y$ in the diagram below.

![Diagram 1](image1)

3. Marcia could walk from $A$ to $B$ along arc $AB$ on the semicircular path, or she can walk along chord $AB$. Diameter $CD$ has length 180 m. How much farther is it to walk along the arc as opposed to the chord?

![Diagram 2](image2)

4. An ant starts at one vertex of a unit cube and walks to the opposite vertex along the surface of the cube. What is the minimum distance the ant can walk?

5. Spot's doghouse has a regular hexagonal base that measures one yard on each side. He is tethered to a vertex with a two-yard rope. What is the area, in square yards, of the region outside the doghouse that Spot can reach?
6. In rectangle $ABCD$, we have $AB = 8$, $BC = 9$, $H$ is on $BC$ with $BH = 6$, $E$ is on $AD$ with $DE = 4$, line $EC$ intersects line $AH$ at $G$, and $F$ is on line $AD$ with $GF \perp AF$. Find the length $GF$.

7. There are two flagpoles, one of height 12 and one of height 16. A rope is connected from the top of each flagpole to the bottom of the other. The ropes intersect at a point $x$ units above the ground. Find $x$. In the accompanying diagram, this is equivalent to finding the length of $EF$.

8. Three spheres are tangent to a plane at the vertices of a triangle and are tangent to each other. Find the radii of these spheres if the sides of the triangle are 6, 8, and 10.

9. Derive a general formula for the volume of the frustum of a cone with bases of radius $R$ and $r$ and height $h$.

Don't look at the next page until you've attempted all the problems!
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The answers to Do You Know Introduction to Geometry are below.

1. (Note that there are many acceptable proofs.) In right triangle $ABC$ with right angle at $A$ we wish to prove $AC^2 + AB^2 = BC^2$. Drop altitude $AD$ to hypotenuse $BC$. $\triangle ABC \sim \triangle DAC \sim \triangle DBA$ giving us $\frac{DC}{AC} = \frac{AC}{BC}$ and $\frac{DB}{AB} = \frac{AB}{BC}$. Now $AC^2 = BC \cdot DC$ and $AB^2 = BC \cdot DB$, so $AC^2 + AB^2 = BC(BC + DB) = BC^2$.

2. $10$

3. $60\pi - 90\sqrt{3}$

4. $\sqrt{5}$

5. $3\pi$

6. $20$

7. $\frac{48}{7}$

8. $r_1 = \frac{12}{5}, r_2 = \frac{15}{4}, r_3 = \frac{20}{3}$

9. $V = \frac{1}{3}\pi h(R^2 + Rr + r^2)$